ON FIXING AND DISTINGUISHING NUMBERS OF TREES

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Abstract. Let G be a graph of order n. A *fixing set* (respectively, *distinguishing coloring*) of G is a subset (resp., partition) of its vertices which is fixed pointwise (resp., setwise) only by the identity automorphism of G. Let F denote the minimum cardinality of a fixing set and D the minimum number of parts, or colors, in a distinguishing coloring of G. Some simple relationships exist between these two common measures of graph symmetry. An instance of a distinguishing coloring of G is a rainbow coloring of a fixing set with a neutral color for the remaining vertices. Hence, $D \leqslant F + 1$. On the other hand, any union of all but one color classes in a distinguishing coloring of G comprises a fixing set, so $F \leqslant (D-1)n/D$. We examine bounds of the latter type for trees, asking: *how large can the fixing number of a D-distinguishable tree of order n be?* For $n \geqslant 3$, we prove sharp upper bounds of $4n/11$ when $D = 2$ and of $(D-1)n/(D+1)$ when $D \geqslant 3$. We then characterize the D-distinguishable trees of radius r ($D, r \geqslant 2$) by way of a family of universal rooted trees T_r^D; a tree of radius r is D-distinguishable if and only if it is a union of branches of T_r^D. Finally, we prove bounds on F and D in terms of the eccentricities of the vertices in a tree. This talk is based on joint work stemming from the 2023 Masamu Advanced Study Institute and Workshop with Peter Dankelmann, Isabel Harris, Paul Horn, K.E. Perry, and Emily Rivett-Carnac.

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